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Solution of Prob. 313 (see p. 164) by Prof. D. V. Wood. — There being no escape of heat due to compression, the law of pressure would be expressed by

$$pv^k = \text{constant} = p'v'^k, \qquad (1)$$

where $p' =$ initial pressure of the atmosphere $=$ 15lbs, nearly, $v' =$ vol. of the cylinder $= al$, if a is the area of the base, $p =$ the pressure within the cylinder when the weight has descended a distance x, $a(l-x) =$ the volume when the pressure is p, $k = 1.408$; then, from (1),

$$p = \frac{p'l^k}{(l-x)^k}, \quad \text{and} \quad \frac{wd^2x}{gdt^2} = w + p'a - \frac{ap'l^k}{(l-x)^k}.$$

Integrating, observing that for $x = 0$, $v = 0$,

$$v^2 = \left(1+\frac{p'a}{w}\right)2gx+\frac{2ap'g}{w(k-1)}\left(\frac{l[l-x]^{k-1}-l^k}{[l-x]^{k-1}}\right).$$

At the end of the downward movement $v = 0$; therefore

$$x(w+pa)(l-x)^{k-1}+\frac{ap'l}{k-1}(l-x)^{k-1} = \frac{ap'l^{k+1}}{k-1},$$

from which x may be found by trial after numerical values have been substituted for the known quantities.

SOLUTIONS OF PROBLEMS IN NUMBER FIVE.

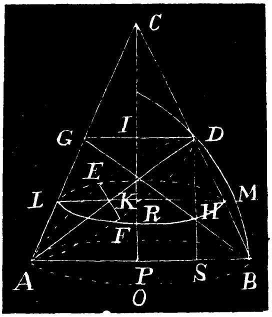
Solutions of problems in No. 5, have been received as follows:

From Prof. W. P. Casey, 320, 321, 323; Prof. J. H. Kershner, 324 and Miscellaneous prob. No. 1 (see p. 149); Octavian L. Mathiot, 320; Prof. D. J. Mc Adam, 323; Prof. E. B. Seitz, 324; Prof. M. C. Woodward, 322; Prof. D. V. Wood, 322; H. Heaton, 322.

320. "The transverse and conjugate axes of an ellipse being given, to find the diameter of the circular base, and the altitude, of the right cone, and where to pass a plane so as to produce the given ellipse."

SOLUTION BY OCTAVIAN L. MATHIOT, BALTIMORE, MD.

Construction. —Draw $AB = 2a$, the given transverse diameter. At P, the centre of AB, erect a perpendicular, and with A as a centre and radius AB, describe a vertical arc in which suppose a point D to be taken such that if BD be produced to meet the perpendicular from P in C, and the triangle PBC be revolved about PC as an axis, a cone will be describd a section of which, having AD for transverse axis, will have the given conjugate axis.



From D to A pass a plane whose cutting edge shall be at right angles with AB and parallel to the base of the cone, producing the ellipse $AFDE$, the transverse axis of which is $AD = AB$. Through K, the centre of AD, pass a plane at right angles to PC and parallel to the base of the cone, producing the circle $LEMF$. Join E and F, the points where the circle intersects the ellipse, EF will be the given conjugate axis $= 2b$, say. Through K, the middle point of EF, draw LM a diameter of the cieele $LEMF$, intersecting PC in R. EF is a chord of the circle $LEMF$.

$$\therefore LK = EK^2 \div KM = b^2 \div KM. \tag{1}$$

Through D, draw DG perpendicular to, and intersecting PC in I, and draw DS parallel to PC.

Calculation.—Since the triangles ALK and AGD are similar, and $AD = 2AK = 2a$, $GD = 2LK$ and $ID = LK$; $\therefore LR = KH = \frac{1}{2}(LK + KM) = (EK^2 + KM^2) \div 2KM$, by substituting for LK from (1), or, because $KM = KD = a$,

$$KH = (a^2 + b^2) \div 2a. \tag{2}$$

We have $HD = \sqrt{(KD^2 - KH^2)} = \sqrt{[(3a^4 - 2a^2b^2 - b^4) \div 4a^2]}$, and $HM = KH - LK = (a^2 - b^2) \div 2a$, and thence, by similar triangles we find $PC = a\sqrt{[(3a^4 - 2a^2b^2 - b^4) \div (a^2 - b^2)]}$, the altitude of the cone.

Again we have $SP = ID = LK = b^2 \div a$, $\therefore AS = (a^2 + b^2) \div a$, and consequently $SD = \sqrt{(AD^2 - AS^2)} = \sqrt{[4a^4 - (a^2 + 2ab + b^2) \div a^2]}$, the altitude of the point D.

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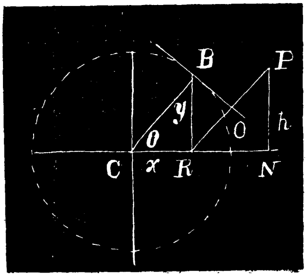
321. "From any point B of the circle $x^2 + y^2 = a^2$, BR is drawn perpendicular to the straight line $x = b$. Find the locus of P, the symmetrical point of R with respect to the tang't at B and its evolute."

SOLUTION BY PROF. W. P. CASEY, SAN FRANCISCO, CAL.



Let $\theta = \angle BCR$, and let p, h be coordinates of P. Then $2y \sin\theta = 2RO = RP$, and $2y \sin^2\theta = RP \sin\theta = h$; $\therefore 4y^2 \sin^4\theta = h^2$. Also $PR \cos\theta = RN$ or $2y \sin\theta \times \cos\theta = RN$; $\therefore 2y \sin\theta \cos\theta + x = p$ and $(2y \sin\theta \cos\theta + x)^2 = p^2$. Therefore $4y^2 \sin^4\theta + 4y^2 \sin^2\theta \cos^2\theta + 4xy \sin\theta \cos\theta + x^2 = p^2 + h^2$, or $4y^2 \sin^2\theta + 4xy \sin\theta \cos\theta + x^2 = p^2 + h^2$, and by assigning numerical values to θ we obtain the locus of P.

322. "The end of a prism is an isosceles triangle, altitude a and base b. The end of this prism is welded to the vertical face of a block of elastic material, the base of the triangular end of the prism being parallel with the horizontal face of the block and projecting a given height h above the block. A given force F_1 is applied at the projecting extremity the direction of which is normal to the vertical face of the block and which is in equilibrium with the forces F_2 and F_3, of tension and pressure, above and below the neutral axis. Find the position of the neutral axis."

SOLUTION BY PROF. M. C. WOODWARD, WASHINGTON UNIV., ST. LOUIS, MO.

Since the prism is in equilibrium, we have

$$F_1+F_2+F_3=0, \tag{1}$$

$$M_1+M_2+M_3=0, \tag{2}$$

in which M is the moment of F about any axis.

Let $x=$ the distance of the required neutral axis from the vertex of the triangular base of the prism. Let r be the distance of any point from the neutral axis; then, supposing the prism to be rigid, we have, for the intensity of the force acting on the prism, $p=-nr$. This is negative, for when r is positive the stress is like F_2, opposite to F_1.

$$F_2=-\frac{bn}{a}\int_0^{a-x-h}(x+r)r\,dr,$$

$$F_3=-\frac{bn}{a}\int_{-x}^{0}(x+r)r\,dr.$$

Letting $s=a-h$, integrating, putting in limits and reducing we have

$$F_2=-\frac{bn}{6a}\left(x^3-3xs^2+2s^3\right),$$

$$F_3=\frac{bn}{6a}x^3.$$

Substituting in (1) we have

$$F_1=\frac{bn}{6a}s^2(2s-3x). \tag{3}$$

Taking the neutral axis as the axis for moments we have

$$M_1=+F_1(a-x),$$

$$M_2=-\frac{bn}{a}\int_0^{s-x}(x+r)r^2dr,$$

$$M_3=-\frac{bn}{a}\int_{-x}^{0}(x+r)r^2dr.$$

Integrating and reducing, we find

$$M_2 = -\frac{bn}{12a}\left(3s^4-8s^3x+6s^2x^2-x^4\right).$$

$$M_3 = -\frac{bn}{12a}x^4.$$

Substituting in (2) we have, after reducing,

$$F_1(a-x) = \frac{bn}{12a}\left(3s^4-8s^3x+6s^2x^2\right). \qquad (4)$$

Combining (3) and (4) we readily get

$$x = \frac{s}{2}\cdot\frac{4a-3s}{3a-2s}.$$

Cor. I. If h is infinite and s finite, we shall have M_1 finite, $F_1=0$ (in order that F_2 and F_3 may be finite), and $x = \frac{2}{3}a$, or we have the neutral axis of a triangular beam exposed only to a bending moment. In this case F_1 is the sum of the two forces of a couple.

Cor. II. If $h = 0$, we have $a = s$, and $x = \frac{1}{2}s$.

Cor. III. If $h = -\frac{1}{3}s$, we have $3a-2s = 0$, $x = \infty$, and the stress is uniform.

Scholium. Regarding the triangular prism as a sector of a solid cylinder, it is easily seen that the general case $h > 0$, is illustrated (approximately) by the tension upon a circular-headed bolt; and the neutral axis becomes a *neutral circle.*

When $h = 0$, we have (still approximately) the action of a nut on a bolt.

When $h = -\frac{1}{3}s$ we have a state of stress very desirable to attain, but very difficult to reach in practice.

[We cannot perceive that the foregoing elegant solution of problem 322 illutrates, even approximately, the tension upon a circular-headed bolt; for the forces acting on the head of the bolt are only *two*, instead of three as in the problem. The head of the bolt being rigid, as the force F_1 is applied at all points of its circumference, there can be no rotation, as is supposed in the problem (322), and therefore no neutral axis. Hence, although we may suppose the bolt elastic, we have no representative of F_3, or rather F_2 and F_3 constitute but one force in the bolt, as they act in the same direction, their *sum* being the equivalent of F_1, instead of their difference as in problem 322. That there can be no neutral axis, or neutral circle, in this case, is evident from the fact that its existence would require a depression in the center which is incompatible with the assumed rigidity of the head of the bolt.—Ed.]

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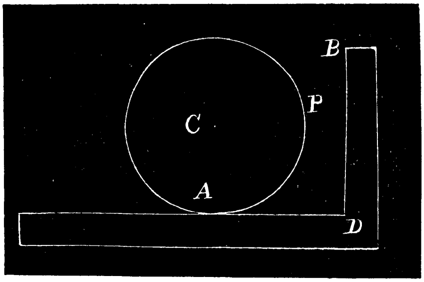
323. "Describe the spiral of Archimedes by continuous motion."

SOLUTION BY PROF. D. J. MC ADAM, WASHINGTON, PA.



Lay a circle, cut from a board or paste board, upon the paper, as *C* in the figure. Lay a square flat upon the paper and make it press against the circumference of the circle. Put a pencil against the arm of the square as at *P*, at a distance from the tang't *AD*, of the radius *AC* of the circle. Roll the square upon the circle and keep the pencil pressed against the p't *P*. The pencil will describe a spiral of Archimedes.

That the curve is the spiral of Archimedes is evident, for there is a constant relation between the angle of rotation and the radius vector *CP*. Also the curve cuts the radial lines from *C* at distances from *C* which increase uniformly by the circumference of the fixed circle. This principle is referred to by Prof. Rankine in his "Machinery and Millwork", page 54.

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324. "Find the envelope of the straight line

$$x \cos \varphi + y \sin \varphi = a (\cos n\varphi)^{\frac{1}{n}}.$$

SOLUTION BY PROF. E. B. SEITZ, KIRKSVILLE, MO.

Differentiating the given equation with respect to φ, we have

$$x \sin \varphi - y \cos \varphi = a \sin n\varphi (\cos n\varphi)^{\frac{1-n}{n}}. \qquad (2)$$

Let $x = r \cos \theta$ and $y = r \sin \theta$, then (1) and (2) become

$$r \cos (\varphi - \theta) = a (\cos n\varphi)^{\frac{1}{n}} \ (3), \quad r \sin (\varphi - \theta) = a \sin n\varphi (\cos n\varphi)^{\frac{1-n}{n}}. \ (4)$$

The square root of the sum of the squares of (3) and (4) is

$$r = a (\cos n\varphi)^{\frac{1-n}{n}}. \qquad (1)$$

Dividing (4) by (3) we have $\tan(\varphi - \theta) = \tan n\varphi$; hence $\varphi - \theta = n\varphi$. or $\varphi = \theta \div (1 - n)$. Substituting this value of φ in (5), we have

$$\cos \frac{n}{n-1} \theta = \left(\frac{a}{r}\right)^{\frac{n}{n-1}},$$

the polar equation of the envelope. When $n = 2$ we have $r^2 \cos 2\theta = a^2$, the equation of the equilateral hyperbola.

$n = -1$, $\quad r \cos^2 \frac{1}{2}\theta = a$, equation of the parabola.
$n = \frac{1}{2}$, $\quad r = a \cos \theta$, " " " circle.
$n = \frac{2}{3}$, $\quad r = a^2 \cos 2\theta$, " " " lemnescate.